

# ONE-DIMENSIONAL FELLER SEMIGROUPS

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In this project, we will focus on one dimensional elliptic operators

$$\mathcal{A}u = qu'' + bu'$$

on  $\mathbb{R}$ , where the coefficients  $q$  and  $b$  are merely continuous on  $\mathbb{R}$  and  $q$  is positive. We will present the Feller theory, developed in [3] at the middle of last century, which allows to characterize the uniqueness of the solution to the elliptic equation

$$\lambda u - \mathcal{A}u = f \in C_b(\mathbb{R}),$$

when  $\lambda$  is positive, in terms of integrability properties of functions obtained from the coefficients  $q$  and  $b$ . Based on this result we will construct the semigroup  $\{T(t)\}$  in  $C_b(\mathbb{R})$  associated to the operator  $\mathcal{A}$ .

Finally, for a particular, but remarkable, class of operators  $\mathcal{A}$  we will characterize the domain of the realization of the operator  $\mathcal{A}$  in  $C_b(\mathbb{R})$ .

## REFERENCES

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- [3] W. Feller, *Diffusion processes in one dimension*, Trans. Amer. Math. Soc. **77** (1954), 1-31.

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