Compactness of semigroups associated with elliptic operators with unbounded coefficients

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Abstract

The aim of this project is to study the compactness in the space $C_b(\mathbb{R}^N)$ of the Feller semigroups $(T(t))_{t\geq 0}$ generated by second order locally uniformly elliptic differential operators

$$A = \sum_{i,j=1}^{N} a_{ij} D_{ij} + \sum_{i=1}^{N} b_i D_i$$

with unbounded coefficients a_{ij} and b_i $(i, j = 1, \dots, N)$ in $\mathbb{R}^N$.

Taking advantage of the construction of the semigroup $T(t)$ provided in [1] we study its compactness property in $C_b(\mathbb{R}^N)$ both when it is generated with the Dirichlet and the maximal domain.

In the latter case we investigate on some consequences of the compactness properties also in connection with the *invariant measure* associated to the semigroup. Many of the results rely on the existence of suitable Lyapunov functions and depend essentially on the growth of the coefficients a_{ij} and b_i at infinity.
All the relevant material is contained in [2].

References

[1] G. Metafune, D. Pallara, M. Wacker, *Feller semigroups on* $\mathbb{R}^N$, Semigroup Forum **65** (2002), 159-205.

[2] G. Metafune, D. Pallara, M. Wacker, *Compactness properties of Feller semigroups*, Studia Mathematica **153** (2002), 179-206.