ON A DIRICHLET PROBLEM INVOLVING AN ORNSTEIN-UHLENBECK OPERATOR

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The aim of this project is to study the Ornstein-Uhlenbeck differential operator

$$L = \frac{1}{2}\Delta + \langle Bx, D\rangle$$

on the half space $H_+ = \left\{x \in \mathbb{R}^N,\ \langle x, e_1\rangle > 0\right\}$, where (e_k) is a fixed orthonormal basis of $\mathbb{R}^N$ and $B = (B_{ij})$ is a nonzero real matrix. In particular, we are concerned with the following second-order Dirichlet problem:

$$\begin{cases} \lambda\varphi(x) - \frac{1}{2}\Delta\varphi(x) - \sum_{ij=1}^{N} B_{ij}x_j D_i\varphi(x) = f(x), & x \in H_+,\ \lambda \geq 0, \\ \varphi(y) = g(y), & y \in \partial H_+. \end{cases} \tag{1}$$

Here f and φ are real uniformly continuous and bounded maps which are defined on the closure of H_+. We study the existence and uniqueness of solutions φ of (1) and prove an optimal Hölder-regularity result for φ. More precisely, we give necessary and sufficient conditions on f and g that ensure the Schauder estimates for the solution φ. For this purpose, we use analytical methods such as semigroups and interpolation theory. We need to apply the Bochner subordination to a class of Markov type semigroups, see [2], which are not C_0-semigroups in uniformly continuous and bounded functions spaces normed with the sup norm. In particular, we extend a theorem in [1] about the analiticity of subordinated semigroups. Moreover, we give explicit formulas for the Poisson kernels when $f = 0$, $\lambda \geq 0$.

The main reference for this project is the paper due to Priola [3].

References

[1] A.S. Carasso and T. Kato, *On subordinated holomorphic semigroups,* Trans. Amer. Math. Soc. 327, 867877, 1991. https://www.jstor.org/stable/2001827

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[3] E. Priola, *On a Dirichlet Problem Involving an OrnsteinUhlenbeck Operator*, Potential Analysis 18, 251-287, 2003. http://dx.doi.org/10.1023/A:1020933325029

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