

INTERPOLATION SPACES

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The Hölder spaces, one of the main protagonists in this year's Internetseminar, are instances of so-called *interpolation spaces* (between spaces of continuously differentiable functions), a fact that has been used several times in the lecture notes in the form of estimates, such as the missing puzzle piece in Lecture 12:

$$\|u\|_{C_b^{1+\alpha}} \leq C_\alpha \|u\|_{C_b^\alpha}^{\frac{1-\alpha}{2}} \|u\|_{C_b^2}^{\frac{1}{2}}, \quad u \in C_b^2(\mathbb{R}^d), \quad \alpha \in (0, 1).$$

The aim of this project is to give an introduction to the abstract theory of interpolation spaces, concentrating on the real interpolation methods, on applications related to elliptic operators on spaces of continuous functions, and on the connection to the theory of strongly continuous semigroups (this latter has been briefly touched upon in Lecture 5). The main source here is the book [1] by A. Lunardi.

As a possible application we shall discuss, if time allows, some Ornstein–Uhlenbeck operators where the diffusion coefficient matrix is degenerate, so that the operator is *not elliptic*, but the drift terms cause an effect that makes the operator behave nicely (the operator will be *hypoelliptic*). We shall prove (optimal) Schauder estimates and arrive at a funny and surprising result being characteristic for hypoelliptic operators, [2].

This project is suited for 3-4 students, and the exact program and content of the project should be formed during the discussions among participants.

REFERENCES

- [1] Alessandra Lunardi, Interpolation theory. Second edition. Appunti. Scuola Normale Superiore di Pisa (Nuova Serie). [Lecture Notes. Scuola Normale Superiore di Pisa (New Series)] Edizioni della Normale, Pisa, 2009. xiv+191 pp.
- [2] Alessandra Lunardi, Schauder estimates for a class of degenerate elliptic and parabolic operators with unbounded coefficients, Rn. Ann. Scuola Norm. Sup. Pisa Cl. Sci. (4) 24 (1997), no. 1, 133–164.