LINEAR PARABOLIC EQUATIONS ON METRIC GRAPHS

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In this project we are going to dip again into the Gauss-Weierstrass semigroup discussed in the ISEM's lectures 4, 5, and 6. We already know how this semigroup looks like and how it acts on functions in $C_b(\mathbb{R})$. However, further one-dimensional settings may be considered: One could think of differential operators on so-called *metric trees*, which can be thought of as a collection of intervals glued together at their endpoints in a network-like fashion, but without cycles. To this aim, suitable transmission conditions have to be imposed in the gluing points.

The participants of this project will introduce these objects formally, learning how to deal with differential operators on such ramified structures and with canonical boundary/transmission conditions. Like in the 4th Lecture of the ISEM, the proof of our main well-posedness result relies essentially upon explicit knowledge of the Gaussian kernel, first derived in [Nic87], with some additional combinatorial flavour.

The project is mainly based on [Cat98], but a few details and a more thorough study of the resolvent operator are necessary. We also refer to [Cat99], where general metric graphs (i.e., graphs that may contain cycles) are considered. While this project is relatively advanced and aimed at graduate students, we are determined to keep it as self-contained as possible.

If wished by the participants, possible extensions of the project include the generalization to slightly different classes of networks (following [Cat99]), the derivation of Schauder estimates or the analysis of Ornstein–Uhlenbeck operators on special classes of trees (both ongoing research topics).

References

[Cat98] C. Cattaneo. The spread of the potential on a homogeneous tree. *Ann. Mat. Pura Appl., IV Ser.*, 175:29-57, 1998.

[Cat99] C. Cattaneo. The spread of the potential on a weighted graph. *Rend. Semin. Mat., Torino*, 57:221-229, 1999.

[Nic87] S. Nicaise. Spectre des réseaux topologiques finis. *Bull. Sci. Math., II. Sér.*, 111:401-413, 1987.

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