20th Internet seminar on "Linear Parabolic equations"

Program

Chapter 1. Maximum principles for parabolic operators

(1) Definition of parabolic operators.
(2) The classical maximum principle for parabolic equations with bounded coefficients in $\mathbb{R}^N$.
(3) The strong maximum principle.
(4) Lyapunov functions.
(5) Maximum principles for solutions to parabolic equations with unbounded coefficients.

Chapter 2. Semigroups of bounded operators

(1) Definition of semigroups of bounded operators.
(2) Strongly continuous semigroups.

Chapter 3. The heat equation in $C_b(\mathbb{R}^N)$ and in $C_b(\mathbb{R}^N_+)$ with homogeneous b.c.

(1) The heat equation in $C_b(\mathbb{R}^N)$ (derivation of the solution via Fourier transform).
(2) Regularity of the solution to the heat equation in $C_b(\mathbb{R}^N)$ and estimates on the solution.
(3) The heat equation in $C_b(\mathbb{R}^N_+)$ (derivation of the solution via reflection).
(4) Regularity of the solution to the heat equation in $C_b(\mathbb{R}^N_+)$ and estimates on the solution.

Chapter 4: the parabolic equation $D_t u = \mathcal{A}u$ in $C_b(\mathbb{R}^N)$

(1) The equation $D_t u - \mathrm{Tr}(QD^2u) = 0$ when Q is constant. Regularity and estimates on the solution. *Simple remark*
(2) Some interpolation estimates.
(3) The continuity method.
(4) The equation $D_t u - \mathrm{Tr}(QD^2u) = 0$ in the general case when Q depends on x; Schauder estimates.
(5) Definition of the associated semigroup and its strong continuity.
(6) A priori interior Schauder estimates (via localization).

Chapter 5: the parabolic equation $D_t u = \mathcal{A}u$ in $C_b(\mathbb{R}^N_+)$ with homogeneous Dirichlet b.c.

(1) The equation $D_t u - \mathrm{Tr}(QD^2u) = 0$ when Q is constant. Regularity and estimates on the solution.
(2) The equation $D_t u - \mathrm{Tr}(QD^2u) = 0$ in the general case when Q depends on x; Schauder estimates.
(3) A priori interior Schauder estimates (via localization).

Chapter 6: the parabolic equation $D_t u = \mathcal{A}u$ in $C_b(\Omega)$ with homogeneous Dirichlet b.c.

(1) Definition of bounded sets of class $C^{2+\alpha}$.
(2) Partition of unity.
(3) The equation $D_t u - \mathrm{Tr}(QD^2u) = 0$ in the general case when Q depends on x; Schauder estimates.
(4) A priori interior Schauder estimates (via localization).

Chapter 7. The Ornstein-Uhlenbeck operator

(1) Definition of the O-U operator.
(2) Rigorous derivation of the O-U semigroup via Fourier transform.
(3) Properties of the O-U semigroup $\{T(t)\}$ in $C_b(\mathbb{R}^N)$ (Invariance of $C_0(\mathbb{R}^N)$, lack of strong continuity and analyticity).
(4) Pointwise and uniform estimates for the spatial derivatives of $T(t)f$.
(5) The invariant measure of the O-U semigroup (definition, necessary and sufficient conditions for its existence, uniqueness of the invariant measure).
(6) Strongly continuity of the O-U semigroup in the L^p-spaces with respect to the Lebesgue measure.

Chapter 8. More general operators with unbounded coefficients in $C_b(\mathbb{R}^N)$

(1) Existence of a solution via approximation with Cauchy-Dirichlet problems in balls.
(2) Uniqueness and non-uniqueness: sufficient conditions implying uniqueness and implying non-uniqueness.
(3) Compactness.