ELLIPTIC PROBLEMS WITH UNBOUNDED COEFFICIENTS

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Consider the differential operator $\mathscr{A}$, defined on smooth functions φ with compact support via

$$\mathscr{A}\varphi = \sum_{i,j=1}^{d} a_{ij}(x)D_{ij}\varphi(x) + \sum_{j=1}^{d} b_j(x)D_j\varphi(x),$$

where the coefficients $a_{ij}, b_j : \mathbb{R}^d \to \mathbb{R}$ are continuous (but not necessarily bounded) for $i,j = 1,\ldots,d$ and the diffusion coefficients a_{ij} are symmetric (i.e. $a_{ij} = a_{ji}$) and satisfy the *ellipticity assumption*

$$\sum_{i,j=1}^{d} a_{ij}\xi_i\xi_j \geq \eta(x)|\xi|^2$$

for all $\xi \in \mathbb{R}^d$ and a function $\eta : \mathbb{R}^d \to (0,\infty)$ satisfying $\inf_{x\in K}\eta(x) > 0$ for every compact set $K \subset \mathbb{R}^d$.

In the lecture notes of the ISEM (see in particular Lecture 14), we have studied *parabolic equations* of the form $\partial_t u - \mathscr{A}u = 0$, where u is a function of time t and the space variable x.

In this project, we turn our attention to the *elliptic equation* $\lambda u - \mathscr{A}u = 0$, where $\lambda > 0$ is a parameter and the sought-after function u depends on the space variable x only. This is the *resolvent equation* of the elliptic operator $\mathscr{A}$. Besides being interesting in its own right, good knowledge of the resolvent equation allows us to use semigroup theory to study the parabolic equation. Consequently, this project is the stepping stone to an alternative approach to parabolic equations for elliptic operators with unbounded coefficients. This, however, will be the content of another project.

Our main reference is the article [1] by Giorgio Metafune, Diego Pallara and Markus Wacker.

References

[1] G. Metafune, D. Pallara, and M. Wacker, *Feller semigroups on* $\mathbb{R}^N$, Semigroup Forum, 65 (2002), pp. 159–205.

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