

The 20th Internet Seminar on
Linear Parabolic Equations

1 Objectives

Our aim is to organize a school where we bring together graduate students, Ph.D students and young researchers from different countries. By doing this we hope to open new areas of research and a fruitful collaboration between operator theorists, and experts in differential and stochastic differential equations.

The main idea is the seminar form of the school, meaning that in a preparatory phase, each participant has to work on a project and then give a talk on the workshop. The projects are coordinated by world known leading experts and the talks of the participating students will be an essential part of the workshop.

The concept of the “Internet Seminar” originates in 1998 when Rainer Nagel (Tübingen) organized the first Internet Seminar. Since then, many schools have been organized in the same spirit and the experience of the previous editions has shown that these schools are much more effective than traditional schools where participants have a much more passive role.

2 The topic of the school

We intend to propose an Internet seminar on parabolic equations mainly on the whole of \mathbb{R}^N with bounded and unbounded coefficients. Clearly, to face operators with unbounded coefficients, one should first set the basis of the classical theory of bounded coefficients, that participants might not have. For this reason, we plan to split the project into two parts. In the first part of the lectures we will illustrate the main results of the classical theory of PDE’s that we need. More specifically, we address the following topics:

- classical maximum principles;
- existence/uniqueness and (local and global) Hölder regularity for solutions to parabolic equations $D_t u - Au = f$ in the whole space and in domains, when A is a uniformly elliptic operator.

We will use the book by Evans: “*Partial differential equations*” and notes written by the organizers. We will start by the Heat equation, where everything is explicit and using the perturbation arguments (e.g., continuity method) we address more general operators.

The second part of the course is devoted to elliptic and parabolic problems with unbounded coefficients. Here, we will follow the book by L. Lorenzi: “*Analytical methods for Kolmogorov equations. Second edition*” (to appear on August 2016) starting from the easiest case of the Ornstein-Uhlenbeck operator to let the participants enter into the field. Everything is explicit with the Ornstein-Uhlenbeck operator, hence, again, everything can be done “by hands”. Based on this easier cases, we then plan to address the general case. We will illustrate the C_b -theory, and address the problem of the uniqueness of the classical solution to the equation $D_t u = Au$ set in the whole \mathbb{R}^N . We will also provide sufficient conditions for the associated semigroup to be or not to be compact in $C_b(\mathbb{R}^N)$.

Some basic facts of Functional analysis which participants should know will be provided in appendices to make the course as much self contained as possible.

3 Preparation: project phase

The preparation starts with a reading course (in the period October 2016-February 2017), which is freely accessible over the internet and actively followed by the registered students and coordinators (usually more than 300 registered participants from all over the world). The phase of the registration will be started at August 20, 2016.

The aim of the lectures is to present the theoretic background which lies behind current ongoing research.

In the project phase (March-May 2017) the participants will form small international groups to work on diverse projects which complement the theory of the lectures and provide some applications of it. Since each group will be formed by students coming from different universities and countries, participants will be required to collaborate using all the tools from internet. Besides this important feature of the seminar form, we think that working on these projects may enable the start of fruitful collaborations for the participants.

4 Final workshop

The main focus of the final workshop in Baronissi (Salerno) (June, 19th-23th 2017) will be on the project presentation by the participants. Each day there will be an invited lecture from 9:00 to 10:00. Potential invited speakers are:

1. Wolfgang Arendt (Universität Ulm);
2. Nikolai Krylov (University of Minnesota);
3. Alessandra Lunardi (Università degli Studi di Parma);
4. Giorgio Metafune (Università del Salento);
5. El Maati Ouhabaz (University of Bordeaux 1);
6. Michael Röckner (Universität Bielefeld);
7. Wilhelm Stannat (Technische Universität Berlin).

Virtual lecturers:

- L. Lorenzi (Parma),
- A. Rhandi (Salerno).